

# **Dynamics of Shallow-Water Internal Waves**

Frank S. Henyey  
Applied Physics Laboratory  
University of Washington  
1013 NE 40th Street  
Seattle, Washington, 98105  
phone: (206) 543-4856 fax: (206) 543-6785 e-mail: [frank@apl.washington.edu](mailto:frank@apl.washington.edu)  
Award #: N00014-95-1-0532  
<http://wavelet.apl.washington.edu/OPD.html>

## **LONG-TERM GOALS**

The long-term goal is to understand the dynamics of the internal wave field on the continental shelf and in straits and fjords. Of interest are both the waves obviously generated by tides and the more random waves present between the tidally generated waves.

## **OBJECTIVES**

We wish to find a model which captures all the main features of the observed tidally generated wave packets. The commonly used Korteweg de Vries equation is known to be adequate at very small amplitudes. We wish to know if the approximations of steadiness and neglect of dissipation on a short time scale, leading to Long's equation, can be made.

## **APPROACH**

Various approximations to the equations of motion for a stratified fluid are investigated, and compared to data from the Solibore and Sill Flows experiment, the Coastal Mixing and Optics experiment, and others. In particular, Long's equation has been examined as a candidate model. Solutions are obtained numerically.

## **WORK COMPLETED**

A number of solutions of Long's equation have been found. Solution of Long's equation with a strong thermocline in a small fraction of the depth required development of a numerical method different from those used with a uniform density gradient.

## **RESULTS**

Long's equation modeling with realistic initial states has solutions that do not resemble the observed waves. In order to more closely resemble the observed waves either unsteadiness or dissipation must be important on short time scales, or there must be trapped fluid regions in the waves.

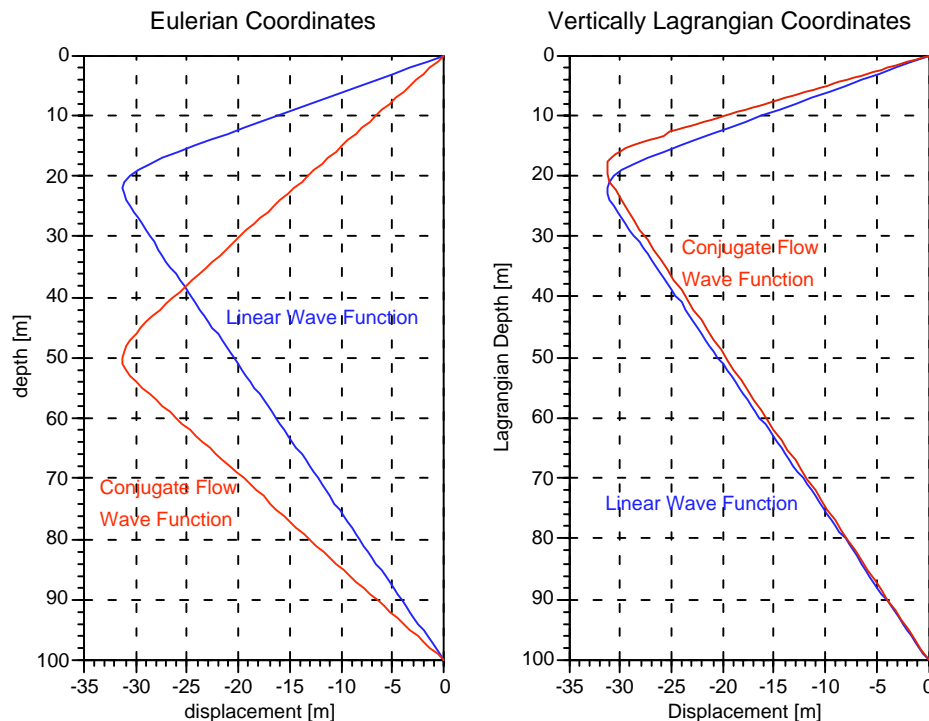
One approach to the modeling follows from a finding of K. Lamb. For conditions that resemble those in which the tidally generated packets are most often seen, he finds that the displacement wave function factorizes, to a good approximation, in a isopycnal, or vertically Lagrangian, coordinate system. For Long's equation, there is a most nonlinear wave, involving transitions to and from a

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>1998</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-1998 to 00-00-1998</b>	
4. TITLE AND SUBTITLE <b>Dynamics of Shallow-Water Internal Waves</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>University of Washington, Applied Physics Laboratory, 1013 NE 40th Street, Seattle, WA, 98195</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES <b>See also ADM002252.</b>					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>3</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

unique "conjugate flow". In the figure, the test of that factorization is applied to model conditions in which there is a 5 m thick thermocline centered at 20 m depth, and the total water depth is 100 m. In the Eulerian coordinate system, the linear and conjugate flow wave functions are very different, and factorization cannot be assumed. In the vertically Lagrangian coordinate system, the wave function very closely resembles the linear wavefunction, so the factorization appears to be a good approximation.

The next step is to use this factorization in a variational ansatz, using the canonical Hamiltonian theory of stratified fluid dynamics. This approach still assumes the neglect of dissipation, so we do not yet know whether it will be successful.

## Comparison of Wave Functions



*The wavefunctions for linear and most nonlinear waves in Long's equation are compared. In the vertically Lagrangian frame they are nearly equal, allowing*

## IMPACT/APPLICATIONS

High amplitude internal waves in coastal oceans are important in a number of ways. They profoundly modify the propagation of sound, and the currents associated with them can affect many marine operations. They may be important in transporting plankton and other substances. The associated non-Gaussian statistics of the coastal internal waves requires a modification to deep-ocean methods of understanding ocean processes and features that depend on internal waves.

## **RELATED PROJECTS**

The Synthetic Aperture Sonar Primer experiment, associated with the Coastal Mixing and Optics program, investigated the effects of coastal internal waves on high-frequency acoustic transmission. The dynamical understanding of the internal waves is crucial for interpreting the acoustics data.

The Solibores and Sill Flows experiment investigated the high-amplitude internal waves in a fjord.

There are a number of projects by various investigators that have measured coastal internal waves in a variety of geographical regions. Close contact between all the investigators

<http://www.whoi.edu/science/AOPE/people/tduda/isww> is essential for making progress on understanding these waves.